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Place your cartoon in the correct Sequence

## Sequences

**A sequence** is an ordered list of numbers.
The **sum** of the terms of a sequence is called a **series.**

- Each number of a sequence is called a **term** (or element) of the sequence.
- A **finite sequence** contains a finite number of terms (you can count them). 1, 4, 7, 10, 13
- An **infinite sequence** contains an infinite number of terms (you cannot count them). 1, 4, 7, 10, 13, . . .
- The terms of a sequence are referred to in the subscripted form shown below, where the **natural number subscript** refers to the location (position) of the term in the sequence.

$$\begin{array}{ccccccc} 1, & 4, & 7, & 10, & 13, & 16, & \ldots \\ a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & \end{array}$$

(If you study computer programming languages such as C, C++, and Java, you will find that the first position in their arrays (sequences) start with a subscript of zero.)

- The **general form of a sequence** is represented: $a_1, a_2, a_3, \ldots a_n, \ldots$
- The **domain of a sequence** consists of the counting numbers 1, 2, 3, 4, ... and the **range** consists of the terms of the sequence.
- The terms in a sequence **may, or may not, have a pattern**, or a related formula. For some sequences, the terms are simply random.

$a_{25} = \underline{\underline{12}}(25)$

**Let's examine some sequences that have patterns:**

Sequences often possess a **definite pattern** that is used to arrive at the sequence's terms. It is often possible to express such patterns as a formula. In the sequence shown at the left, an **explicit formula** may be:

$a_n = 12n$

where $n$ represents the term's position in the sequence.

$$\overset{a_1}{\underset{(1)}{12}},\ \overset{a_2}{\underset{12(2)}{24}},\ \overset{a_3}{\underset{12(3)}{36}},\ \ldots$$

$a_n = 12n$

$a_1 = 12(1) = 12$

$a_2 = 12(2) = 24$

$\left(\frac{(n+1)}{n}\right)4$

**Examples:**

**1.** Write the first three terms of the sequence whose $n^{th}$ term is given by the explicit formula:

$$a_n = 2n - 1$$

$a_n = X + 7$

$a_1 = 1 + 2 \cdot 3$

$a_1 = 2(1) - 1 = 1$

$a_2 = 2(2) - 1 = 3$

$a_3 = 2(3) - 1 = 5$

**ANSWER:** Remember that $n$ is a natural number (starting with $n = 1$).

$$a_1 = 2(1) - 1 = 1$$
$$a_2 = 2(2) - 1 = 3$$
$$a_3 = 2(3) - 1 = 5$$

Notice that $n$ is replaced with the number of the term you are trying to find.

**2.** Find the 5th and 10th terms of the sequence whose $n$th term is given by:

$$a_n = \frac{n}{n+1}$$

$$a_n = \frac{n}{n-1} \qquad a_1 = \frac{1}{1-1} = \frac{1}{0} \text{ und}$$

$$a_5 = \frac{5}{5-1} = \frac{5}{4}$$

$$a_{10} = \frac{10}{10-1} = \frac{10}{9}$$

**ANSWER:** Remember that $n$ corresponds to the location of the term. Use $n = 5$ and $n = 10$.

$$a_5 = \frac{5}{5-1} = \frac{5}{6}$$

$$a_{10} = \frac{10}{10+1} = \frac{10}{11}$$

**3.** Write an explicit formula for the $n^{\text{th}}$ term of a sequence of negative even integers starting with -2.

$-2, -4, -6, -8, \ldots$

$a_n = 2n - 1$

$1, 3, 5, 7, 9, \ldots$

$\boxed{a_n = -2n}$

$a_1 = -2(1) = -2$

$a_2 = -2(2) = -4$

**ANSWER:** Get a visual of the terms. -2, -4, -6, -8, ...

Compare the terms to the numbers associated with their locations and look for a attern.

| Notation | Location | Term |
|---|---|---|
| $a_1$ | 1 | -2 |
| $a_2$ | 2 | -4 |
| $a_3$ | 3 | -6 |
| $a_4$ | 4 | -8 |

Look for a pattern. In this example, each term can be found by multiplying the location number by -2.
A formula could be:

$$a_n = -2n$$

**4.** Find the first 4 terms of the sequence

$$a_n = (-1)^n (n^2 + 3)$$

$$a_1 = (-1)^1 (1^2 + 3) = -4$$
$$a_2 = (-1)^2 (2^2 + 3) = +7$$
$$a_3 = (-1)^3 (3^2 + 3) = -12$$
$$a_4 = (-1)^4 (4^2 + 3) = +19$$

Notice how the terms are alternating signs between negativ and positive.

**Keep this pattern in mind** (involving powers of -1) when asked to write formulas for sequences.

$(-1)^n (n^2 + 3)$ yields -4, 7, -12, 19, ...

$(-1)^{n+1} (n^2 + 3)$ yields 4, -7, 12, -19, ...

**Answer the following questions pertaining to sequences.**

1.

$a_n = 3n$ term of the sequence

2nd Term

$a_2 = 3(2) = 6$

**Choose:**

3
6
9

2.

$a_n = n^2 - 1$ ms of the sequence:

$a_1 =$

**Choose:**

0, 3, 8
3, 8, 15
1, 2, 3

3.

$a_n = n(n+2)$ the sequence

$12(12+2) = 12 \cdot 14 = 168$

$a_{12} = 168$

**Choose:**

146
168
196

4.

Find the 8th term of the sequence

$a_n = \left(\frac{1}{2}\right)^n = \quad a_8 = \left(\frac{1}{2}\right)^8 = \frac{1}{256}$

**Choose:**

$1/256$
$1/64$
$1/16$

$\frac{1^8}{2^8} =$

**5.**

**Choose:**

$\cancel{a_n = 4n+1}$

$\cancel{a_n = n+4}$

$a_n = 4n$

Write a formula for the sequence
4, 8, 12, 16, 20, ...

$a_1, a_2, a_3, \ldots$

---

**6.**

**Choose:**

196

~~-225~~

225

Find the 1 $(-1)^{n-1} n^2$ nce

$a_n = (-1)^{n-1} n^2 = (-1)^{14} (15)^2 = (+1)(225)$

---

**7.**

**Choose:**

-144

~~144~~

$-e^{144}$

$e^{144}$

Find the 12th term of the sequence $a_n = (-1)^{n+1} e^{n^2}$

$(-1)^{12+1} e^{(12)^2} = (-1)^{13} e^{144} = (-1) e^{144}$

# DO NOW

Not on your Paper

1 Each number in a sequence is known as a term.

True

False

*Select the correct answer.*

**Geometric Sequences**
**and Series**

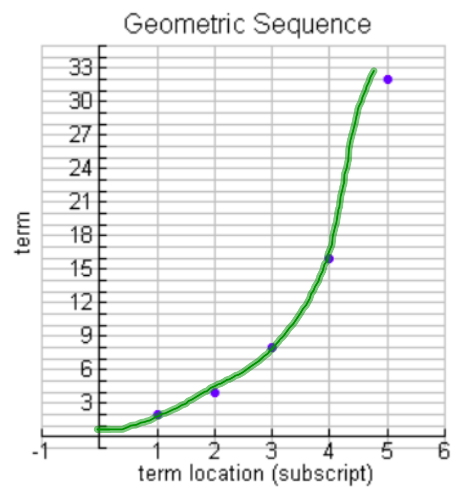


**A sequence** is an ordered list of numbers.
The **sum** of the terms of a sequence is called a **series.**
While some sequences are simply random values,
other sequences have a **definite pattern** that is used to arrive at the sequence's terms.
Two such sequences are the **arithmetic** and **geometric** sequences. Let's investigate the geometric sequence.

| | |
|---|---|
| **Geometric Sequences** |   |

If a sequence of values follows a pattern of **multiplyinga fixed amount** (not zero) times each term to arrive at the following term, it is referred to as a **geometric sequence**. The number multiplied each time is constant (always the same).

The fixed amount multiplied is called the **common ratio, *r***, referring to the fact that the ratio (fraction) of the second term to the first term yields this common multiple. To find the common ratio, divide the second term by the first term.




Notice the **non-linear nature** of the scatter plot of the terms of a geometric sequence. The domain consists of the counting numbers 1, 2, 3, 4, ... and the range consists of the terms of the sequence. While the $x$ value increases by a constant value of one, the y value increases by multiples of two (for this graph).

**Examples:**

| Geometric Sequence | Common Ratio, $r$ | |
|---|---|---|
| $\frac{10}{5}=2$   $a_1$ $a_2$ 5, 10, 20, 40, ... | $r = 2$ | **multiply** each term by 2 to arrive at the next term or...**divide** $a_2$ by $a_1$ to find the common ratio, 2. |
| $22/-11$   -11, 22, -44, 88, ... | $r = -2$ | **multiply** each term by -2 to arrive at the next term .or...**divide** $a_2$ by $a_1$ to find the common ratio, -2. |
| $\frac{8}{3}/4$   $4, \frac{8}{3}, \frac{16}{9}, \frac{32}{27}, \frac{64}{81}, \ldots$ | $r = \frac{2}{3}$ | **multiply** each term by 2/3 to arrive at the next term or...**divide** $a_2$ by $a_1$ to find the common ratio, 2/3. |

**Formulas used with geometric sequences and geometric series:**

$n, r, a_1$

$17, 2, 5$

$a_{17} = 5 \cdot 2^{16}$

To **find any term**
of a **geometric sequence:**

$$a_n = a_1 \bullet r^{n-1}$$

where $a_1$ is the first term of the sequence,
$r$ is the common ratio,
$n$ is the number of the term to find.

*Note:* $a_1$ is often simply referred to as $a$.

To find the **sum of a certain number of terms**
of a **geometric sequence:**

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

where $S_n$ is the sum of $n$ terms ($n^{th}$ **partial sum**),
$a_1$ is the first term, $r$ is the common ration.

$$S_4 = \frac{5(1-2^4)}{1-2}$$

$$\frac{5(1-16)}{-1}$$

$$\frac{5(-15)}{-1}$$

$$\frac{-75}{-1} = 75$$

**Examples:**

| Question | Answer |
| --- | --- |
| **1.** Find the common ratio for the sequence $a_1\ a_2$ $6, -3, \frac{3}{2}, -\frac{3}{4}, \ldots$ | |

$$r = \frac{a_2}{a_1} = \frac{-3}{6} = -0.5 = -\frac{1}{2}$$

| | |
|---|---|
| **2.** Find the common ratio for the sequence given by the formula $$a_n = 5(3)^{n-1}$$ | |

$$a_n = a_1 \cdot r^{n-1}$$

common ratio.

**3.** Find the 7[th] term of the sequence

2, 6, 18, 54, ...

$r = \frac{6}{2} = 3$

$a_n = a_1 \cdot r^{n-1}$

$a_7 = 2 \cdot 3^{7-1} = 2 \cdot 3^6$

$= 2 \cdot 729 = 1458$

**4.** Find the $11^{th}$ term of the sequence

$$1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \ldots$$

$a_{11} = 1 \cdot (-0.5)^{11-1} = 1 * 0.5^{10}$

$r = {}^{-}\frac{1/2}{1} = {}^{-}0.5$

$= 9.765E{-4}$

$= 0.000976$

$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

**5.** Find $a_8$ for the sequence
$$0.5, 3.5, 24.5, 171.5, \ldots$$

$$r = \frac{3.5}{0.5} = 7$$

$$a_8 = 0.5 * 7^{8-1}$$

$$= 0.5 * 7^7 = 411{,}771.5$$

**6.** Evaluate using a formula:

Sigma ← $\sum_{k=1}^{5} 3^k$

means add all terms

$a_1 : 3^1 = 3$ $\quad \frac{9}{3} = 3$

$a_2 = 3^2 = 9$

$S_n = \frac{a_1(1-r^n)}{1-r} = \frac{3(1-3^5)}{1-3} = \frac{3(1-243)}{1-3}$

$\frac{3(-242)}{-2} = 363$

**6.** Examine the summation

$$\sum_{k=1}^{5} 3^k = 3^1 + 3^2 + 3^3 + 3^4 + 3^5$$

This is a geometric series with a common ratio of 3.

$n = 5;\ a_1 = 3, r = 3$

$$S_5 = \frac{3(1-3^5)}{1-3} = \frac{-726}{-2} = 363$$

**7.** Find the sum of the first 8 terms of the sequence

$-5, 15, -45, 135, \ldots$

$$r = \frac{15}{-5} = -3$$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_8 = \frac{-5(1-(-3)^8)}{1-(-3)} = \frac{-5(1-6561)}{4} = \frac{-5(-656}{4}$$

$$= 8{,}200$$

**7.** The word "sum" indicates a need for the sum formula.

$n = 8;\ a_1 = -5, r = -3$

$$S_8 = \frac{-5(1-(-3)^8)}{1-(-3)}$$

$$S_8 = \frac{-5(1-6561)}{4} = \frac{32800}{4} = 8200$$

## Check Your Understanding

2 What are the first five terms of a geometric sequence that begins with 3 and has a common ratio of 2?

A 3, 6, 12, 24, 48.

B 3, 6, 9, 12, 15.

C 3, 5, 7, 9, 11.

D 3, 4, 5, 6, 7.

*Select the correct answer.*

**8.** The third term of a geometric sequence is 3 and the sixth term is 1/9. Find the first term.

**9.** A ball is dropped from a height of 8 feet. The ball bounces to 80% of its previous height with each bounce. How high (*to the nearest tenth of a foot*) does the ball bounce on the fifth bounce?

Teacher's Notes

## Arithmetic Sequences

In an arithmetic sequence, each succeeding term differs from the proceeding term by the same number.

In an arithmetic sequence, the difference between two subsequent terms is known as the common difference.

The $n^{th}$ term of an arithmetic sequence can be calculated by the formula:

$$a^n = a^1 + (n-1)d$$

where:
n = the number of terms you want to find
$a^1$ = the first term
d = common difference

Example 1

| Arithmetic Sequence | Common Difference |
| --- | --- |
| 1, 3, 5, 7, 9, 11, …….. | 2 |
| 2, 7, 12, 17, 22, 27, ……. | 5 |

Example 2

Identify the sixth term of an arithmetic sequence that begins with 5 and common difference is 9.

$a^n = a^1 + (n-1)d$

$a^6 = 5 + (6-1)9$

$= 5 + 5 \times 9$

$= 5 + 45$

$= 50$

Hence, the sixth term is 50.

Teacher's Notes

# Activity

Identify the common difference between the terms and complete the sequence.

| Sequence | Common Difference |
|---|---|
| 2, 4, 6, ... | |
| 2, 5, 8, ... | |
| 5, 13, 21, ... | |

## Check Your Understanding

3 What is the eighth term of an arithmetic sequence that begins with 6 and has a common difference of 10?

A 66

B 76

C 86

D 96

*Select the correct answer.*

## Check Your Understanding

4 What is the fifth term of a geometric sequence that begins with 3 and has a common difference of 3?

A 324

B 108

C 36

D 12

*Select the correct answer.*

## Check Your Understanding

5 What are the first five terms of a geometric sequence that begins with 3 and has a common ratio of 2?

A 3, 6, 12, 24, 48.

B 3, 6, 9, 12, 15.

C 3, 5, 7, 9, 11.

D 3, 4, 5, 6, 7.

*Select the correct answer.*

## Check Your Understanding

6 What is the tenth term of an arithmetic sequence that begins with 2 and has a common difference of 5?

A 32

B 37

C 42

D 47

*Select the correct answer.*

## Check Your Understanding

7 What is the common ratio of the geometric sequence -3, 12, -48, 192, -768, …..?

A -3

B 3

C -4

D 4

*Select the correct answer.*

## Check Your Understanding

8 What is the tenth term of an arithmetic sequence that begins with 2 and has a common difference of 5?

A 32

B 37

C 42

D 47

*Select the correct answer.*

## Check Your Understanding

9 What is the common ratio of the geometric sequence -3, 12, -48, 192, -768, …..?

A -3

B 3

C -4

D 4

*Select the correct answer.*

## Check Your Understanding

10 What is the fifth term of a geometric sequence that begins with 3 and has a common difference of 3?

A 324

B 108

C 36

D 12

*Select the correct answer.*